5.2 - Sum and Difference Identities

Warmup

Find the exact value

1) $\cos(45° - 30°)$

$$\frac{\sqrt{6}+\sqrt{2}}{4}$$

2) $\cos(120° - 45°)$

$$\frac{\sqrt{6}-\sqrt{2}}{4}$$

3) $\sin(45° - 30°)$

$$\frac{\sqrt{6}-\sqrt{2}}{4}$$

4) $\sin(60° - 45°)$

$$\frac{\sqrt{6}-\sqrt{2}}{4}$$

5.1 – Trigonometric Identities

Practice - Find the missing term

1) $\dfrac{\cos x}{1-\tan x}+\dfrac{\sin x}{1-\cot x}=\sin x+???$ $\quad ??? = \cos x$

2) $\csc x-\cot x=\dfrac{???}{1+\cos}$ $\quad ??? = \sin x$

Chapter 5
Trigonometric Identities

1. Trigonometric Identities
2. Sum and Difference Identities
3. Double-Angle Identities
4. **Half-Angle Identities**
5. Product-to-Sum Identities

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

$$\sin\left(\frac{\theta}{2}\right) \quad \text{and} \quad \cos\left(\frac{\theta}{2}\right)$$

Start with the Cosine double-angle identity

$$\cos(2A) = 1 - 2\sin^2 A$$

$$2\sin^2 A = 1 - \cos(2A)$$

$$\sin^2 A = \frac{1 - \cos(2A)}{2}$$

$$\sin A = \pm\sqrt{\frac{1 - \cos(2A)}{2}} \quad \text{Substitute } A = \frac{\theta}{2}$$

$$\boxed{\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

$$\sin\left(\frac{\theta}{2}\right) \quad \text{and} \quad \cos\left(\frac{\theta}{2}\right)$$

Start with the other Cosine double-angle identity

$$\cos(2A) = 2\cos^2 A - 1$$

$$2\cos^2 A = \cos(2A) + 1$$

$$\cos^2 A = \frac{1 + \cos(2A)}{2}$$

$$\cos A = \pm\sqrt{\frac{1 + \cos(2A)}{2}} \quad \text{Substitute } A = \frac{\theta}{2}$$

$$\boxed{\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

Start with the Tangent identity

$$\tan\left(\frac{A}{2}\right) = \frac{\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\pm\sqrt{\frac{1-\cos A}{2}}}{\pm\sqrt{\frac{1+\cos A}{2}}}$$

$$\boxed{\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

Another way to derive a Tangent identity

$$1-\cos A = 2\sin^2\left(\frac{A}{2}\right) \quad \text{and} \quad \sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$

$$\frac{1-\cos A}{\sin A} = \frac{2\sin^2\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \tan\frac{A}{2}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

5.4 - Half-Angle Identities

Deriving the Half Angle Identities

Yet another way to derive a Tangent identity

$1 + \cos A = 2\cos^2\left(\frac{A}{2}\right)$ and $\sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$

$$\frac{\sin A}{1+\cos A} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \tan\frac{A}{2}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

5.4 - Half-Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A}$$

Practice - Use the half-angle identities

1. $\cos\left(\frac{5\pi}{8}\right)$

$$-\frac{\sqrt{2-\sqrt{2}}}{2}$$

2. $\tan\left(\frac{11\pi}{12}\right)$

$$-2+\sqrt{3} \quad \text{or} \quad -\sqrt{7-4\sqrt{3}}$$

3. $\cos x = \frac{1}{5}$, *find* $\cos\left(\frac{x}{2}\right)$

$$\pm\frac{\sqrt{15}}{5}$$

5.4 - Half-Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A}$$

Practice - Find the exact value, x is in 1st quadrant

$$\cos(x) = \frac{5}{13}$$

1. $\cos\left(\frac{x}{2}\right)$

$\frac{3\sqrt{13}}{13}$

2. $\sin\left(\frac{x}{4}\right)$

$\frac{\sqrt{338-78\sqrt{13}}}{26}$