Chapter 1
Right Triangle Trigonometry

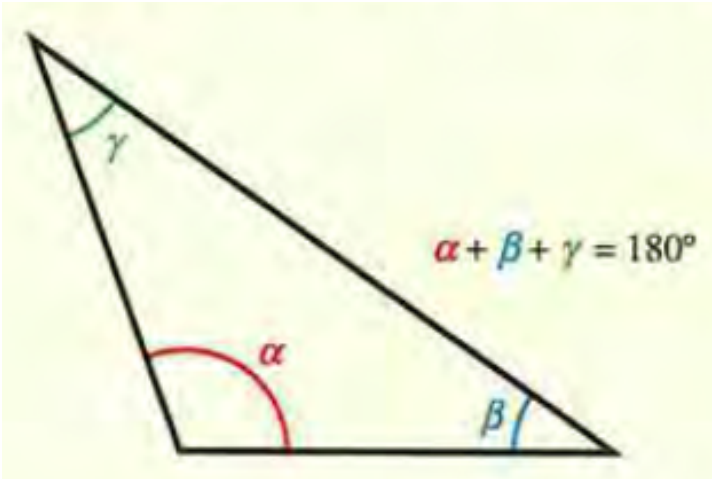
1. **Angles, Degrees, and Triangles**
2. Similar Triangles
3. Right Triangle Ratios
4. Evaluating Trigonometric Functions
5. Solving Right Triangles

Basics of Triangles

1.1 - Angles, Degrees, and Triangles

Triangle Sum Theorem

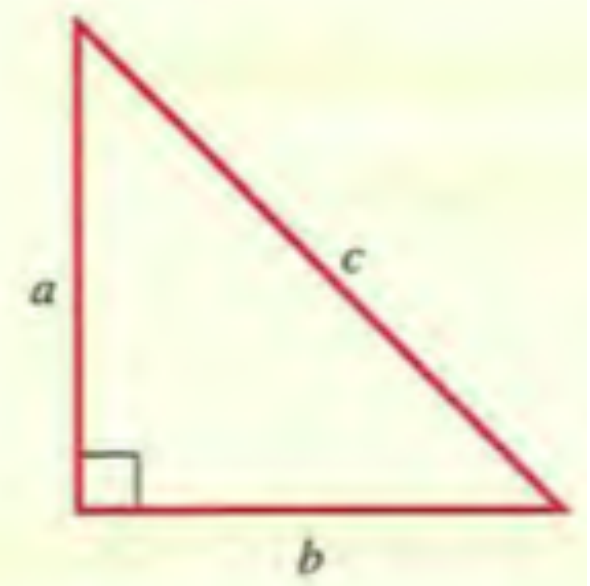
The sum of the interior angles is 180˚.



Pythagorean Theorem

In any right triangle

$$a^2 + b^2 = c^2$$



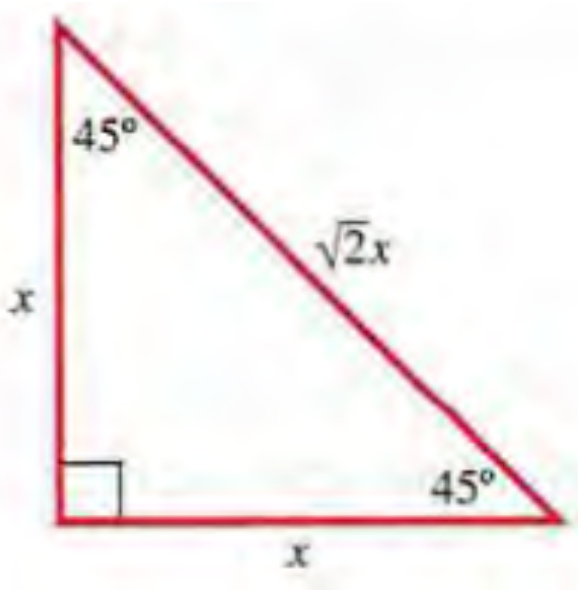
1.1 - Angles, Degrees, and Triangles

Special Right Triangles

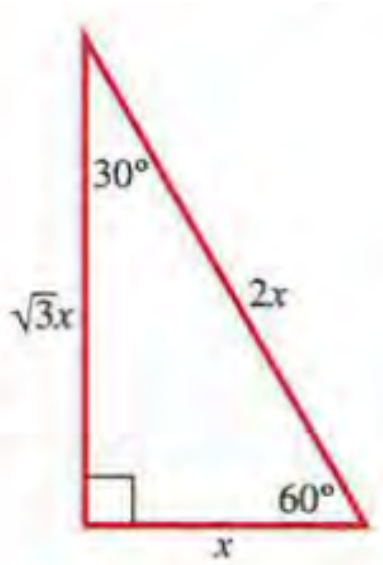
- **Pythagorean Triples** - Integer numbers that satisfy the Pythagorean Theorem.

$3^2 + 4^2 = 5^2$ $\quad 5^2 + 12^2 = 13^2$ $\quad 8^2 + 15^2 = 17^2$

- **45˚-45˚-90˚**



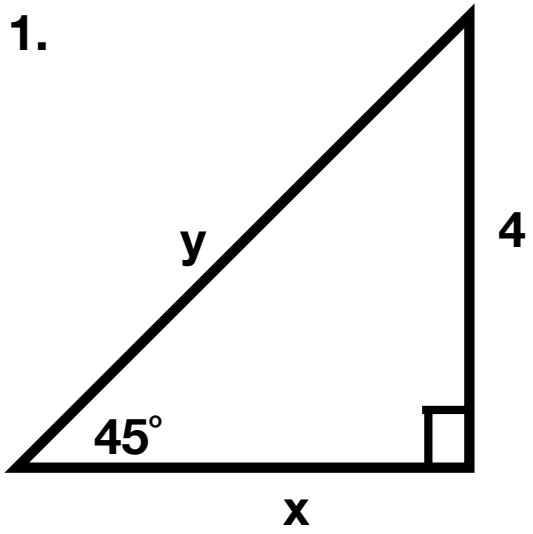
- **30˚-60˚-90˚**



1.1 - Angles, Degrees, and Triangles

Practice

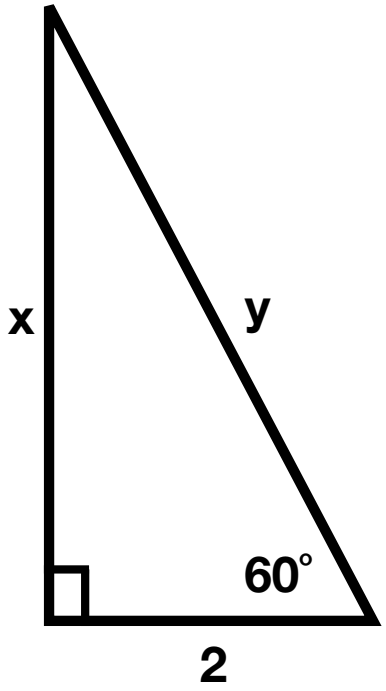
1.



x = 4

y = $4\sqrt{2}$

2.



x = $2\sqrt{3}$

y = 4

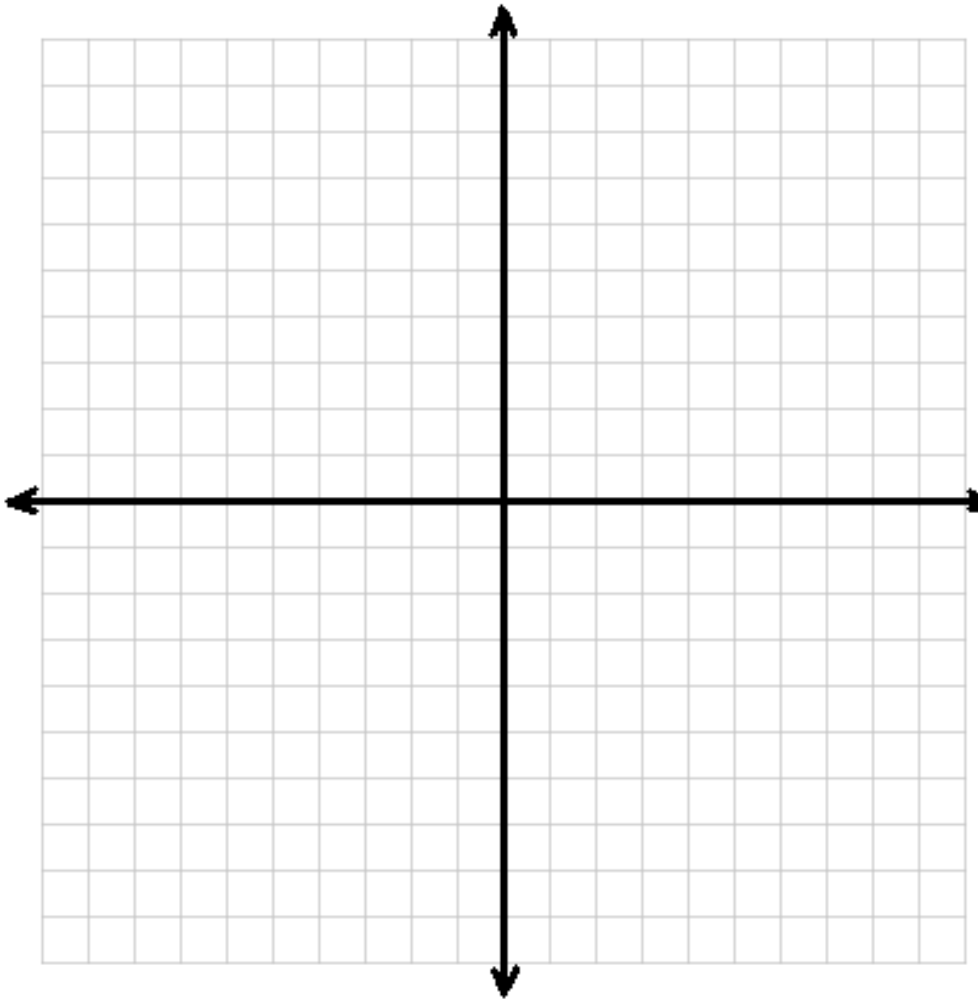
1.1 - Angles, Degrees, and Triangles

Real World Problem

London Eye

The wheel takes 30 minutes to make a complete rotation. What will it rotate in 12 minutes?

144°



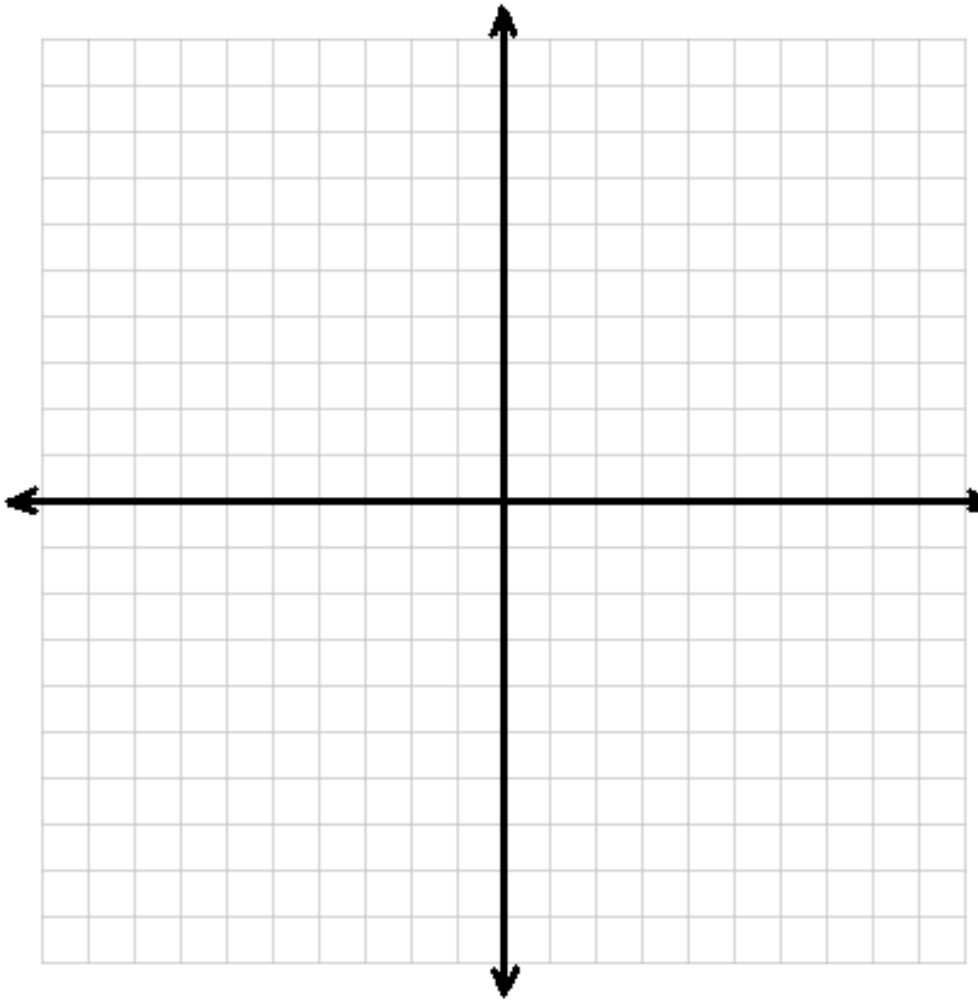
1.1 - Angles, Degrees, and Triangles

Real World Problem

Clock

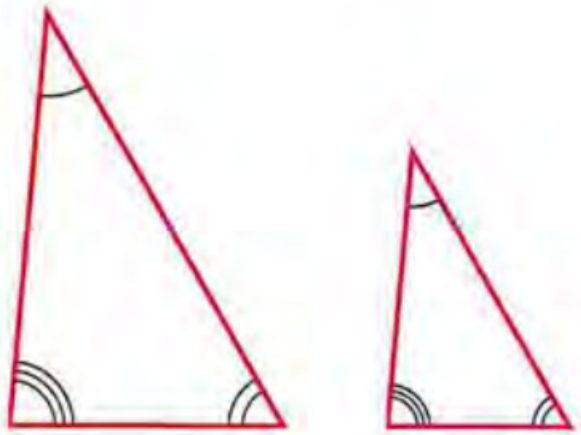
The hands of a clock are at 10:40. What acute angle do the big and little hands make?

80°

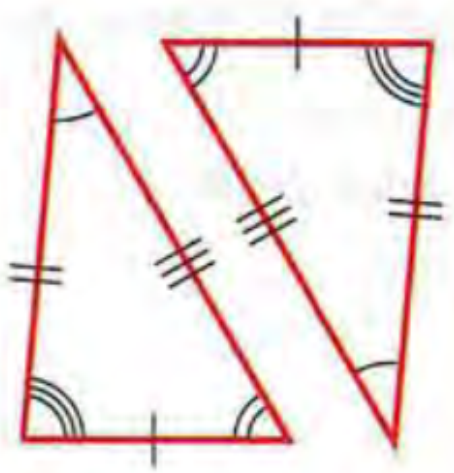


1.2 - Similar Triangles

Similar triangles - corresponding angle measures equal and side measures proportional



Congruent triangles - equal corresponding angle measures and equal corresponding side measures



1.2 - Similar Triangles

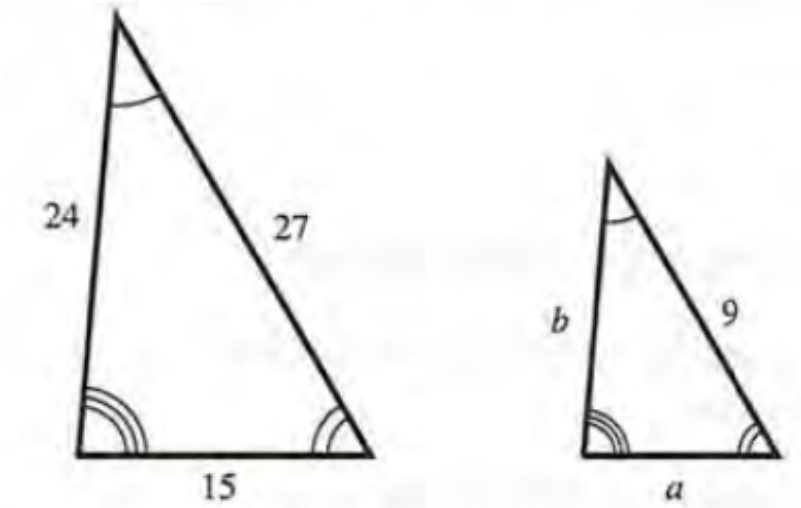
Cape Hatteras Lighthouse - The lighthouse is the tallest in North America. If a 5 ft woman casts a 1 1/2 foot shadow and the lighthouse casts a 48 ft shadow, approximately how tall is the lighthouse?

160 ft

1.2 - Similar Triangles

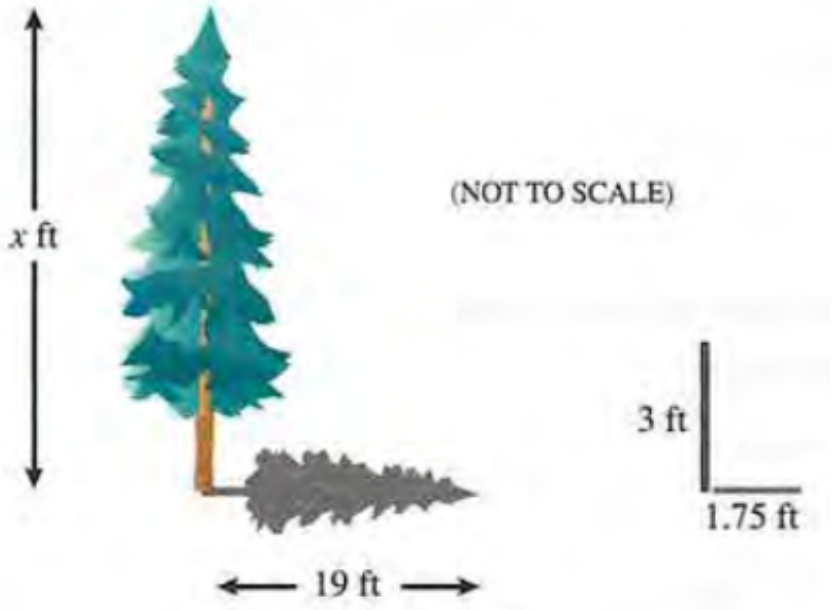
Practice

a. Find the unknown sides



a = 5
b = 8

b. Billy wants to find the height of a tree. He measures the shadow of the tree. He also measures the height and shadow of a stake in the ground.

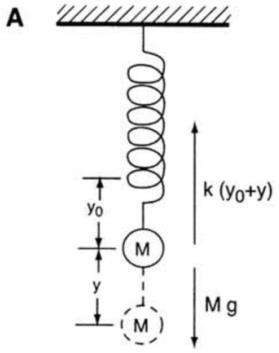


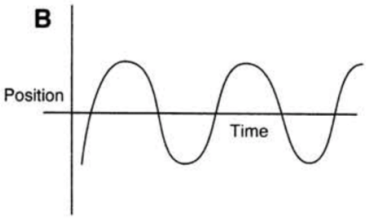
$x \approx 32.57\ ft$

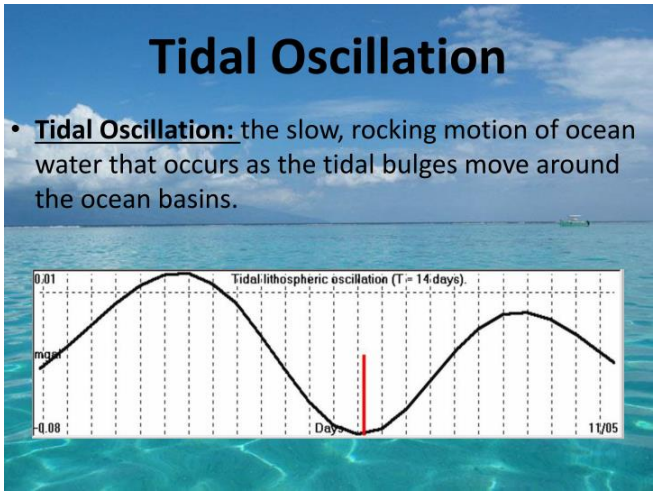
Trigonometric Ratios

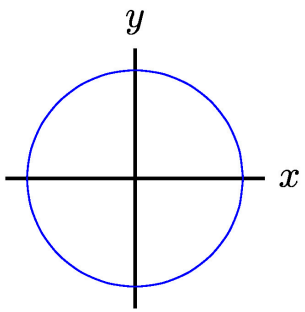
1.3 - Right Triangle Ratios

- Uses of Trigonometry
  - Oscillation
  - Recurring events
  - Parametric equations
  - Polar rose curves
  - AC current
  - …

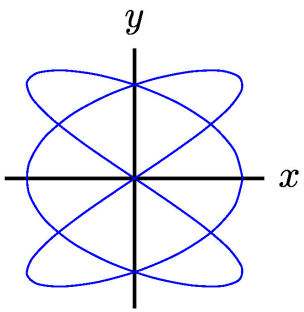




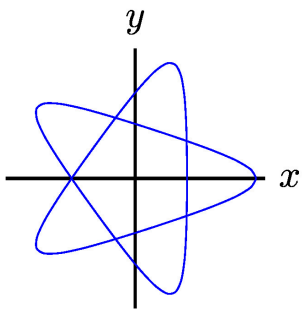




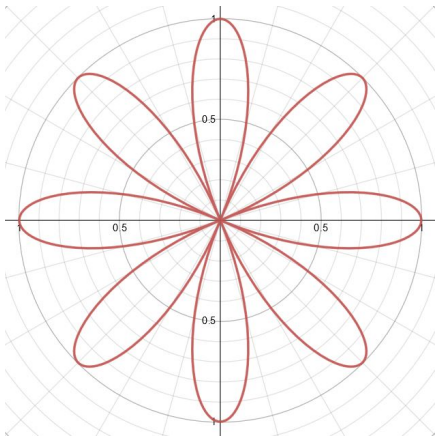
$x(t) = \cos(t)$
$y(t) = \sin(t)$



$x(t) = 3\cos(t)$
$y(t) = 2\sin(t)$



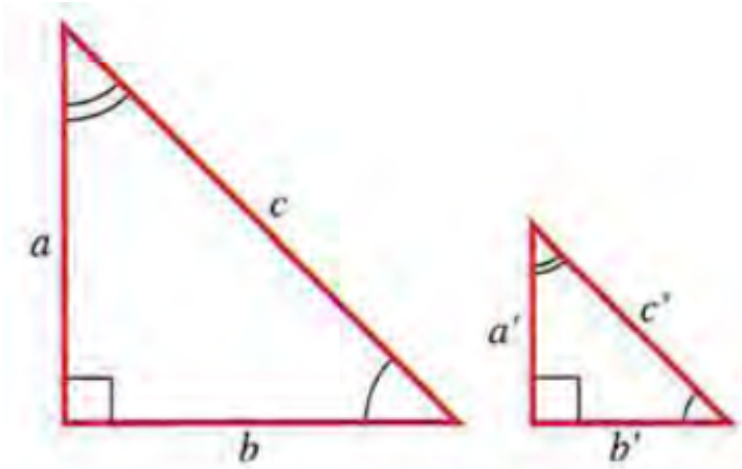
$x(t) = 2\cos(x) + 5\cos(\frac{2}{3}x)$
$y(t) = 2\sin(x) + 5\sin(\frac{2}{3}x)$



1.3 - Right Triangle Ratios

Trigonometry is based on:

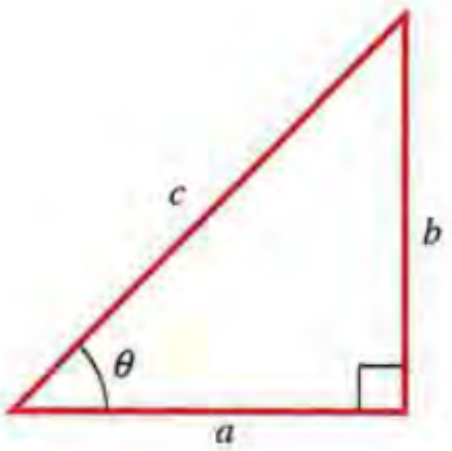
Similar right triangles - corresponding angle measures equal and side measures proportional



$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} \longrightarrow \frac{a}{b} = \frac{a'}{b'} \quad \text{and ...} \quad \frac{b}{c} = \frac{b'}{c'}$$

1.3 - Right Triangle Ratios

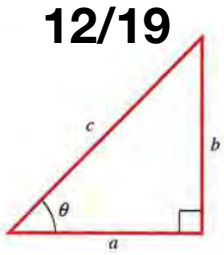
Trigonometric ratios - based on the ratio of the sides of a right triangle and one of the *acute angles*.



$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

$$\sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c} \qquad \tan\theta = \frac{b}{a}$$

1.3 - Right Triangle Ratios



Main identities - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Setting Your Calculator

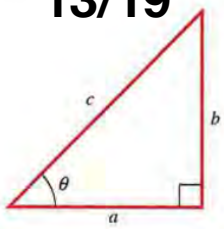
We are using degrees - “D” or “DEG” on calculator

Another setting is radians - “R” or “RAD” on calculator

FYI, $360^\circ = 2\pi$ radians

DO NOT USE!!! (yet)

1.3 - Right Triangle Ratios



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$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

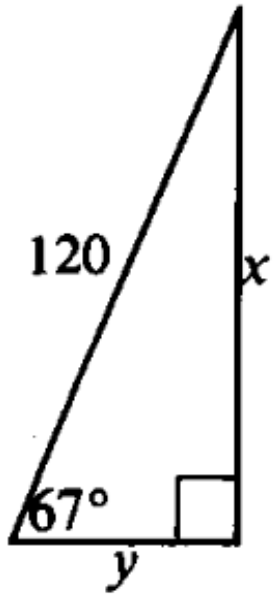
Find the values x and y to the nearest hundredth.

$$\sin 67 = \frac{x}{120} \qquad \cos 67 = \frac{y}{120}$$

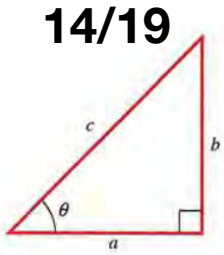
$$x = 120\sin 67 \qquad y = 120\cos 67$$

$$x \approx 120(0.9205) \qquad y \approx 120(0.3907)$$

$$x \approx 110.46 \qquad y \approx 46.89$$



1.3 - Right Triangle Ratios

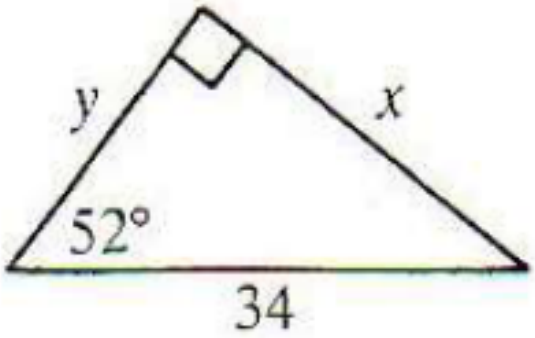


Main identities - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Practice: Find the values of x and y to the nearest hundredth.

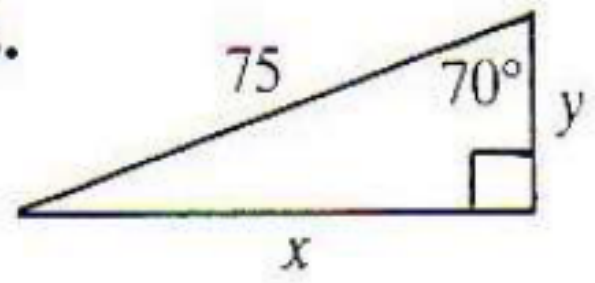
11.



$x \approx 26.79$

$y \approx 20.93$

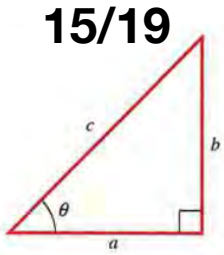
12.



$x \approx 70.48$

$y \approx 25.65$

1.3 - Right Triangle Ratios



Main identities - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).

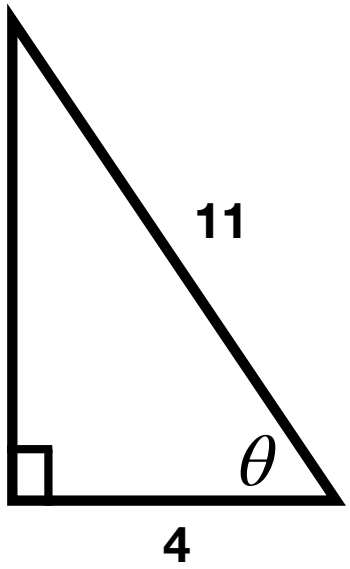
$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

What about finding θ?

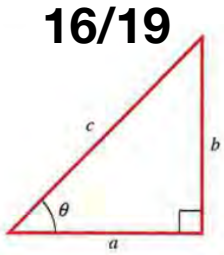
$$\cos\theta = \frac{4}{11}$$

$$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{4}{11}\right)$$

$$\theta \approx 68.68^{\circ}$$



1.3 - Right Triangle Ratios

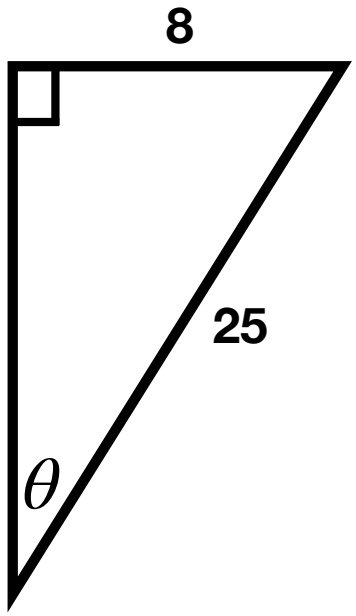


Main identities - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Practice: Find θ.

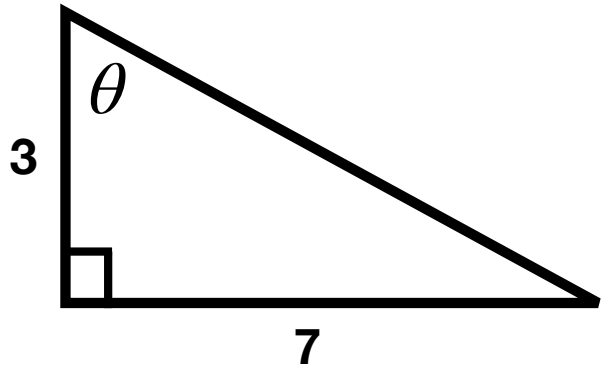
1.



$\sin\theta = \frac{8}{25}$

$\theta \approx 18.66^\circ$

2.



$\tan\theta = \frac{7}{3}$

$\theta \approx 66.8^\circ$

1.3 - Right Triangle Ratios

Main identities - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).

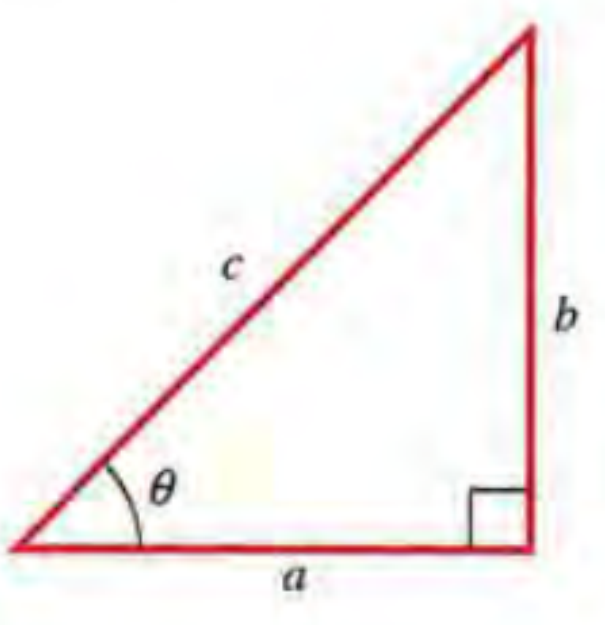
$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Cosecant, secant, and cotangent are reciprocals of …

$$\csc\theta = \frac{1}{\sin\theta} = \frac{c}{b}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{c}{a}$$

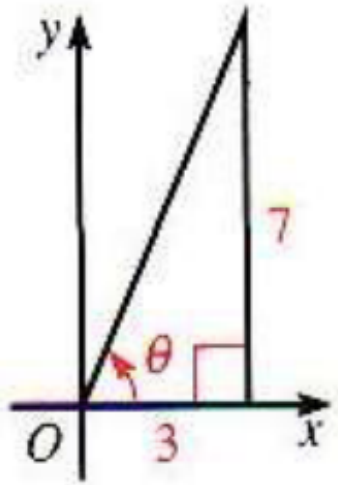
$$\cot\theta = \frac{1}{\tan\theta} = \frac{a}{b}$$



1.3 - Right Triangle Ratios

Practice: Give the values of the six trigonometric functions of θ.

1.

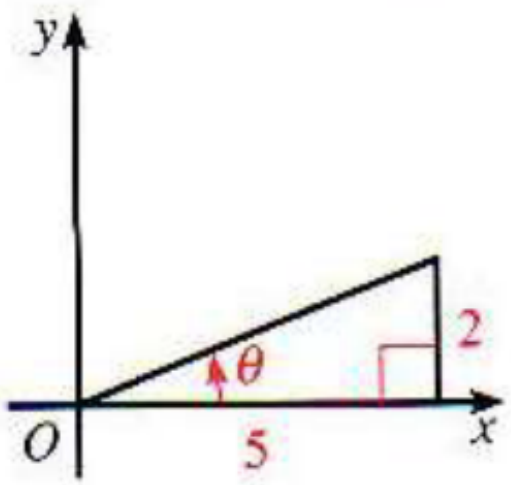


$$\sin\theta = \frac{7\sqrt{58}}{58} \qquad \csc\theta = \frac{\sqrt{58}}{7}$$

$$\cos\theta = \frac{3\sqrt{58}}{58} \qquad \sec\theta = \frac{\sqrt{58}}{3}$$

$$\tan\theta = \frac{7}{3} \qquad \cot\theta = \frac{3}{7}$$

2.



$$\sin\theta = \frac{2\sqrt{29}}{29} \qquad \csc\theta = \frac{\sqrt{29}}{2}$$

$$\cos\theta = \frac{5\sqrt{29}}{29} \qquad \sec\theta = \frac{\sqrt{29}}{5}$$

$$\tan\theta = \frac{2}{5} \qquad \cot\theta = \frac{5}{2}$$

1.3 - Right Triangle Ratios

Practice: Ishaan and Sania are planning to hike to the top of a hill. If θ is the angle formed by the hill and the ground such that $\sec\theta = 1.75$, find $\sin\theta$.

$\sin\theta = 0.821$