8.3 - Analyzing Geometric Sequences and Series

Warmup

Find the indicated sum for each series.

Geometric

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

1) $1 + 2 + 4 + \ldots;\ S_8$

255

2) $3 + 6 + 12 + \ldots;\ S_7$

381

3) $4 + 12 + 36 + \ldots;\ S_6$

1456

4) $7 - 35 + 175 - \ldots;\ S_5$

3647

5) $7 + 70 + 700 + \ldots;\ S_7$

7,777,777

6) $11 + 33 + 99 + \ldots;\ S_8$

36,080

8.5 - Using Recursive Rules with Sequences

An **explicit rule** gives a_n as a function of the term's position number n in a sequence.

A **recursive rule** give the beginning term and a recursive equation.

Explicit Definition

$$a_n = a_1 + (n-1)d$$

$$a_n = 5 + (n-1)4$$

$$a_n = 4n + 1$$

Recursive Definition

$$a_1 = 5$$

$$a_n = a_{n-1} + 4$$

8.5 - Using Recursive Rules with Sequences

Recursive Equations for Arithmetic and Geometric Sequences

Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference

Geometric Sequence

$a_n = r \cdot a_{n-1}$, where r is the common ratio

Write a recursive rule for the sequence.

$a)\ 2, 14, 98, 686, 4802, \ldots$

$a_1 = 2;\ a_n = 7a_{n-1}$

$b)\ 19, 13, 7, 1, -5, \ldots$

$a_1 = 19;\ a_n = a_{n-1} - 6$

8.5 - Using Recursive Rules with Sequences

Find the first 6 terms.

$a_1 = 1$

$a_2 = 1$

$a_n = a_{n-2} + 2 \cdot a_{n-1}$

$a_3 = ??$

$a_3 = a_1 + 2 \cdot a_2$

$a_3 = 1 + 2 \cdot 1$

$a_3 = 3$

$a_4 = ??$

$a_4 = a_2 + 2 \cdot a_3$

$a_4 = 1 + 2 \cdot 3$

$a_4 = 7$

1, 1, 3, 7, 17, 41

8.5 - Using Recursive Rules with Sequences

Find the first 6 terms.

$$a_1 = 1$$
$$a_2 = 1$$
$$a_n = a_{n-2} + 2 \cdot a_{n-1}$$

1, 1, 3, 7, 17, 41

Practice - Find the first six terms

1) $a_1 = 8$

$$a_n = a_{n-1} - 1$$

8, 7, 6, 5, 4, 3

2) $a_1 = 1, a_2 = 2$

$$a_n = (a_{n-2})^2 + a_{n-1}$$

1, 2, 3, 7, 16, 65

8.5 - Using Recursive Rules with Sequences

Write a recursive rule for the sequence.

1
2
4
8
16 . . .

$a_1 = 1,\ a_n = 2 \cdot a_{n-1}$

Two base cases

1
1
2
3
5
8
13 . . .

$a_1 = 1,\ a_2 = 1,\ a_n = a_{n-1} + a_{n-2}$

8.5 - Using Recursive Rules with Sequences

Practice - Write a recursive rule for the sequence.

$a)\ a_n = 17 - 4n$

$a_1 = 13,\ a_n = a_{n-1} - 4$

$b)\ \frac{5}{6}, \frac{7}{12}, \frac{1}{3}, \frac{1}{12}, \ldots$

$a_1 = \frac{5}{6},\ a_n = a_{n-1} - \frac{1}{4}$

8.5 - Using Recursive Rules with Sequences

Fibonacci Sequence

$1, 1, 2, 3, 5, 8, 13, \ldots$ $\quad a_1 = 1 \quad a_2 = 1$

$$a_n = a_{n-2} + a_{n-1}$$

Practice - Write a recursive formula (hard)

Each has two base cases

a) $1, 1, 3, 7, 17, 41, 99, \ldots$

$a_1 = 1,\ a_2 = 1$

$a_n = 2 \cdot a_{n-1} + a_{n-2}$

b) $1, 2, 3, 7, 22, 155, \ldots$

$a_1 = 1,\ a_2 = 2$

$a_n = a_{n-1} \cdot a_{n-2} + 1$