Conics - Ellipses

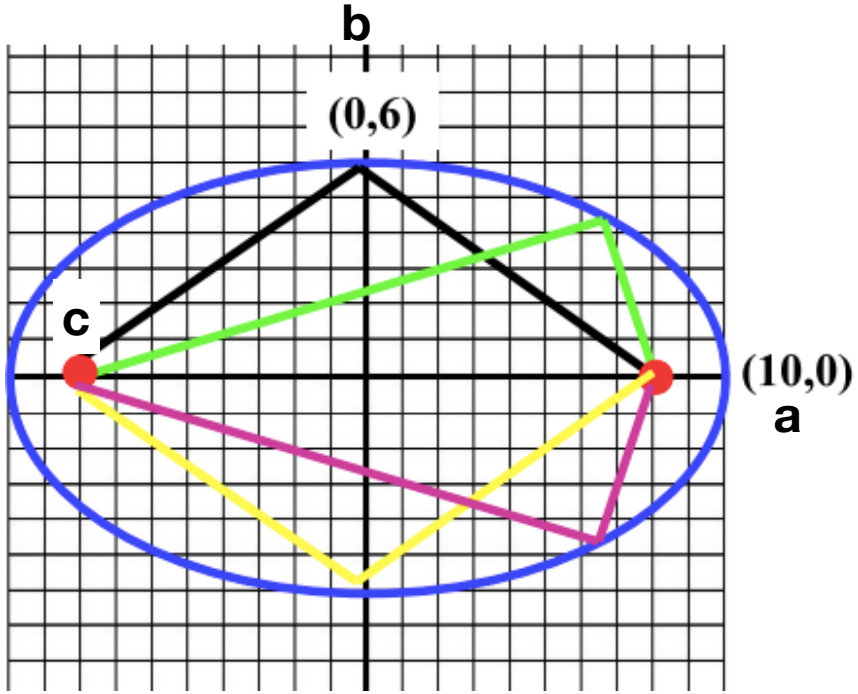
Why are they called conics?

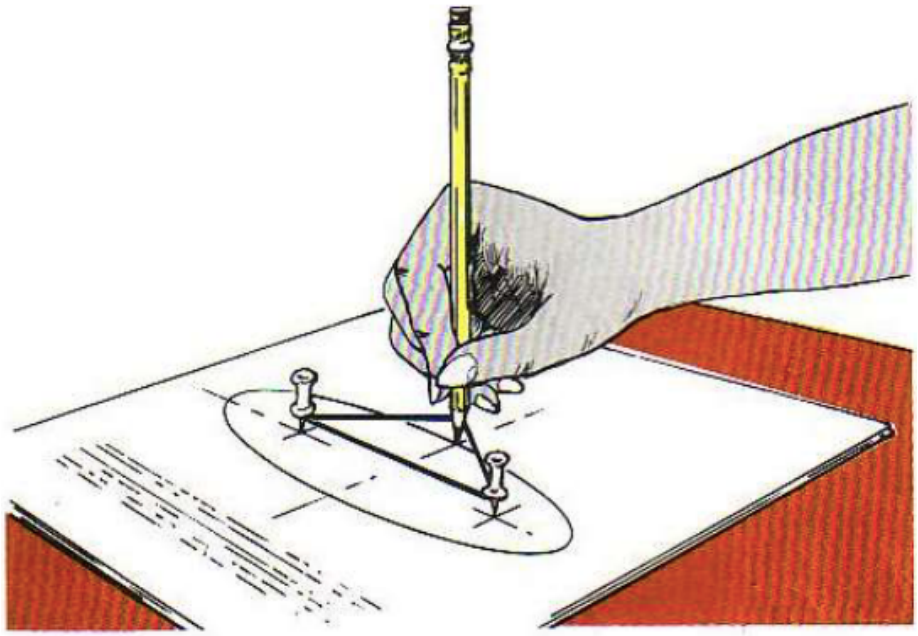
Circle *Parabola* *Ellipse* *Hyperbola*

What about a line?

Conics - Ellipses

How do I draw an ellipse?





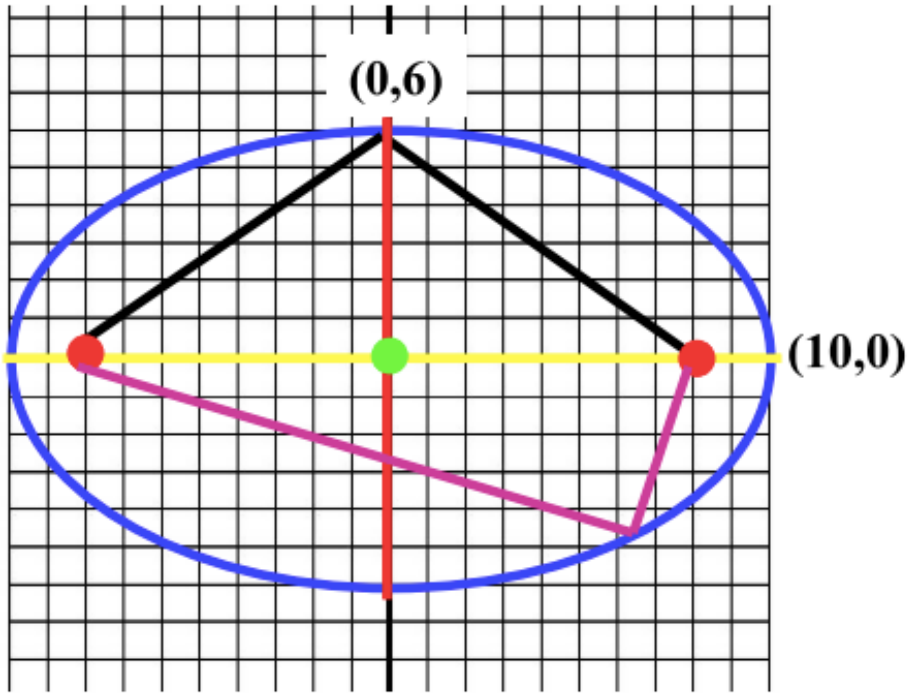
$$36x^2 + 100y^2 = 3600$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

What is the distance from b to c?

Conics - Ellipses

Vocabulary

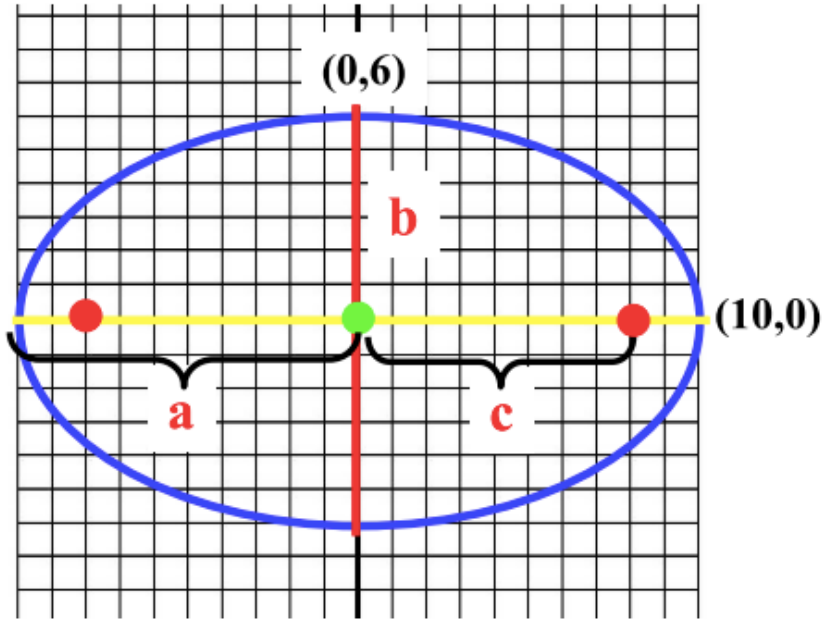


- Foci - red points
- Center - green point
- Major Axis - yellow chord (long)
- Minor Axis - red chord (short)
- Focal Radii - length of black or purple lines

What is the focal radii of the ellipse? 2a = 20

Conics - Ellipses

What is the equation for an ellipse?



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

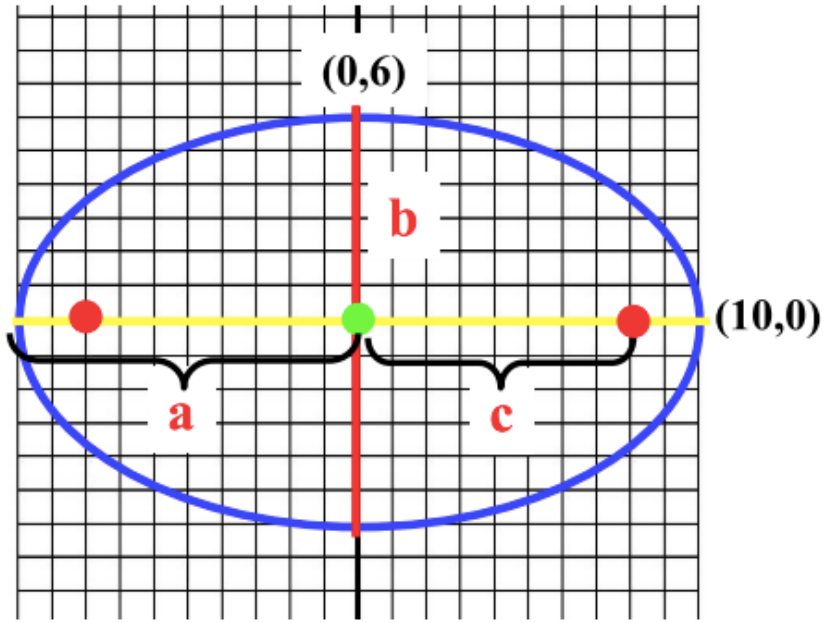
$$\frac{x^2}{10^2} + \frac{y^2}{6^2} = 1$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

What is the focal radius? $2a = 20$

Conics - Ellipses

What is the equation for an ellipse?



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{10^2}+\frac{y^2}{6^2}=1$$

$$\frac{x^2}{100}+\frac{y^2}{36}=1$$

What is the focal radius? $2a = 20$

Write the equation and find focal radius (FR)

1) a=4, b=2

$\frac{x^2}{16}+\frac{y^2}{4}=1;\ FR=8$

2) a=5, b=3

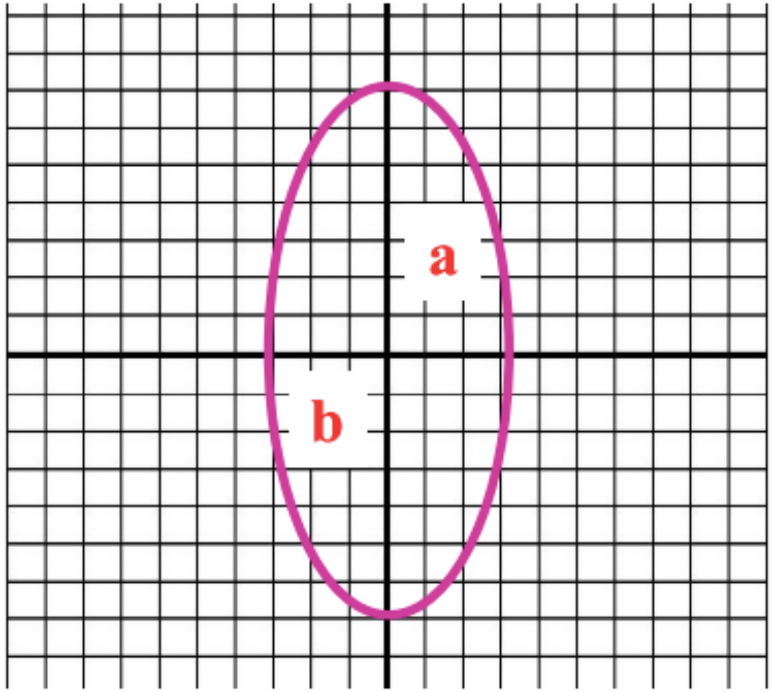
$\frac{x^2}{25}+\frac{y^2}{9}=1;\ FR=10$

3) $a=\sqrt{26}$, $b=2\sqrt{5}$

$\frac{x^2}{26}+\frac{y^2}{20}=1;\ FR=2\sqrt{26}$

Conics - Ellipses

What happens if we switch axes?



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{49} = 1$$

Conics - Ellipses

What if you were given the following equation.

$$4x^2 + y^2 = 64 \quad \rightarrow \quad \frac{4x^2 + y^2 = 64}{64}$$

$$\frac{x^2}{16} + \frac{y^2}{64} = 1$$

What about $4x^2 + 4y^2 = 16$

Write the ellipse equation.

1. $2x^2 + 3y^2 = 6$

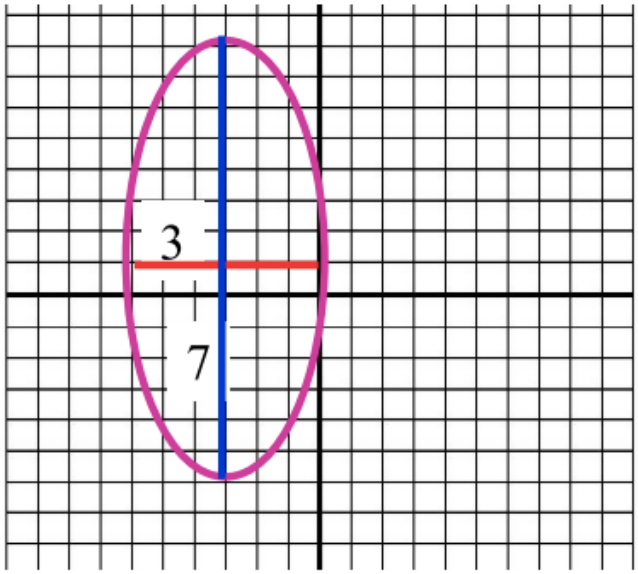
$$\frac{x^2}{3} + \frac{y^2}{2} = 1$$

2. $12x^2 - 36 = -3y^2$

$$\frac{x^2}{3} + \frac{y^2}{12} = 1$$

Conics - Ellipses

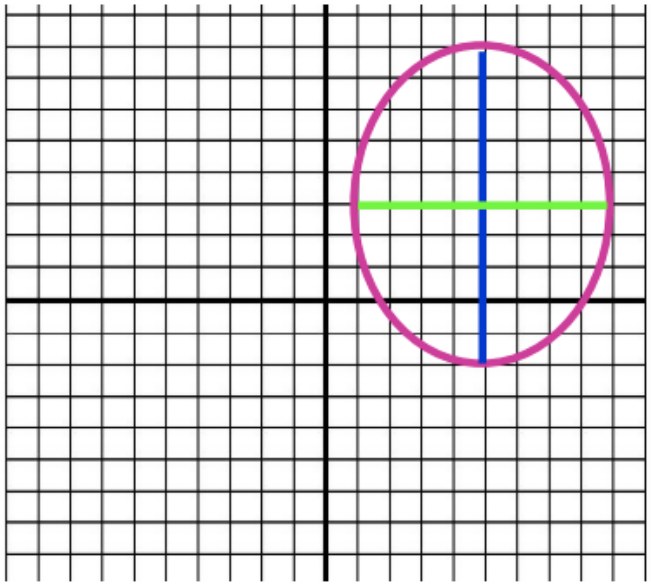
What if the center is not at the origin? C(-3, 1)



$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\frac{(x-h)^2}{3^2}+\frac{(y-k)^2}{7^2}=1$$

$$\frac{(x+3)^2}{9}+\frac{(y-1)^2}{49}=1$$



Write the equation

$$\frac{(x-5)^2}{16}+\frac{(y-3)^2}{25}=1$$

Conics - Ellipses

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$4x^2+y^2+24x-10y+45=0$$

$$(4x^2+24x)+(y^2-10y)=-45$$

$$4(x^2+6x+?)+(y^2-10y+?)=-45+?$$

$$4(x^2+6x+9)+(y^2-10y+25)=-45+36+25$$

$$4(x+3)^2+(y-5)^2=16$$

$$\frac{(x+3)^2}{4}+\frac{(y-5)^2}{16}=1$$

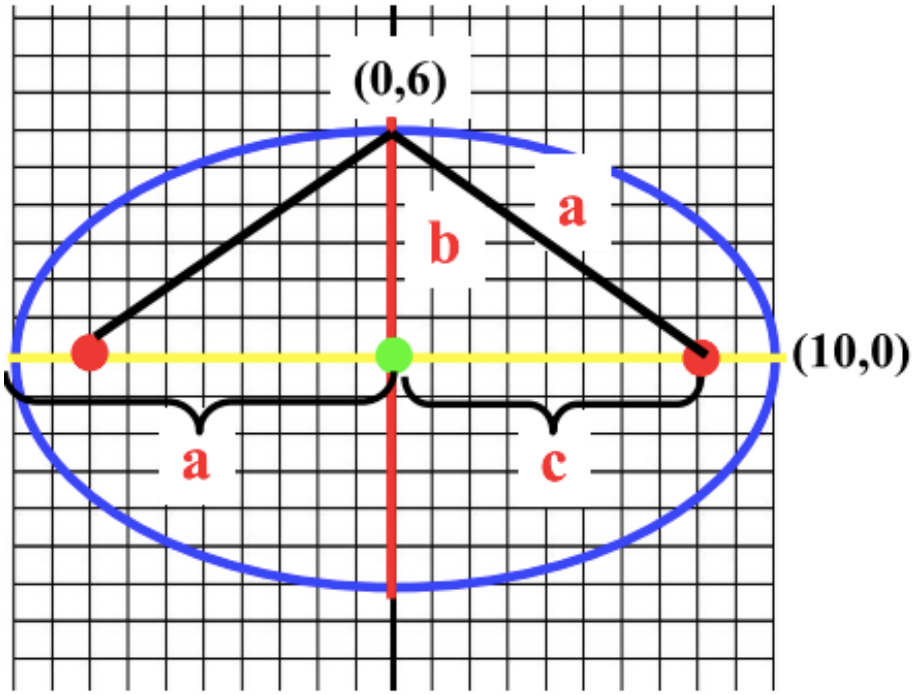
Write the equation

$$9x^2+4y^2-18x+16y=11$$

$$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1$$

Conics - Ellipses

Is there a relationship between a, b, and c?



YES!! $a^2 = b^2 + c^2$

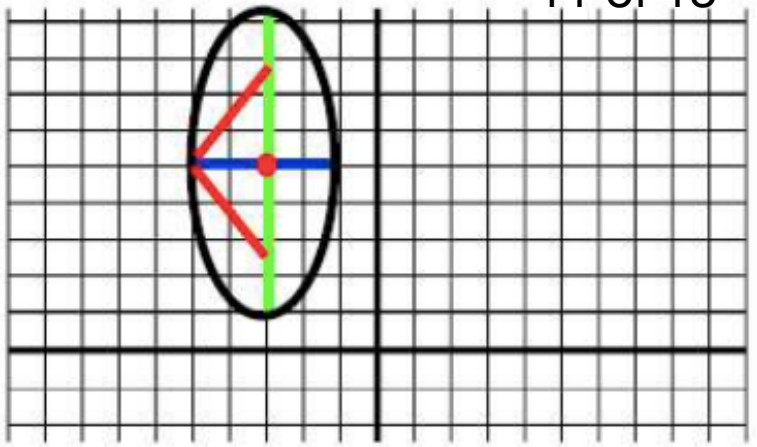
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

c is the distance from the center to each focus.

Conics - Ellipses

Graph and find the foci.

$$\frac{(x+3)^2}{4}+\frac{(y-5)^2}{16}=1$$



Center (-3, 5)

$a = 4$

$b = 2$

$a^2 = b^2 + c^2$

$4^2 = 2^2 + c^2$

$12 = c^2$

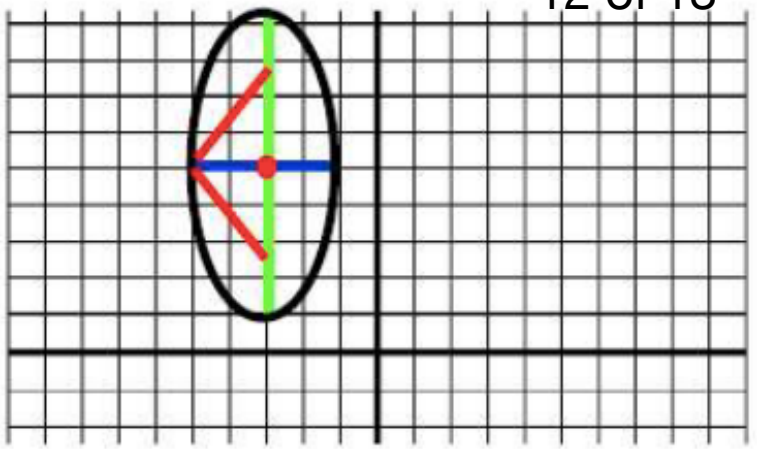
$c = 2\sqrt{3}$

$f_1 = (-3,5-2\sqrt{3})$ $f_2 = (-3,5+2\sqrt{3})$

Conics - Ellipses

Graph and find the foci.

$$\frac{(x+3)^2}{4}+\frac{(y-5)^2}{16}=1$$



Center (-3, 5)

$a=4$

$b=2$

$a^2=b^2+c^2$

$4^2=2^2+c^2$

$12=c^2$

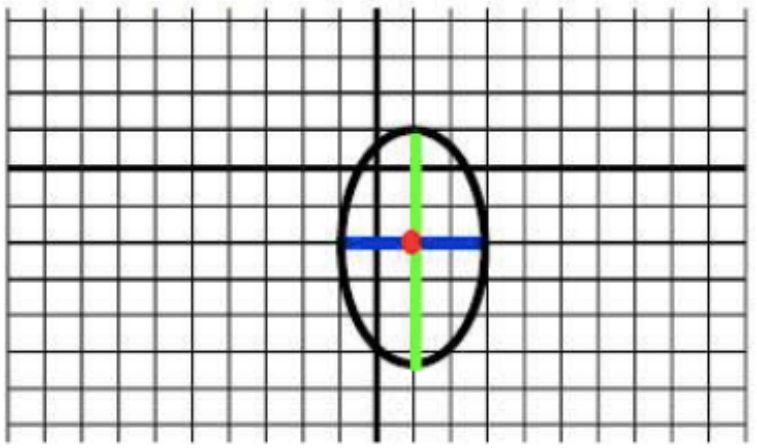
$c=2\sqrt{3}$

$f_1=(-3,5-2\sqrt{3})$ $f_2=(-3,5+2\sqrt{3})$

Graph and find the foci.

$$9x^2+4y^2-18x+16y=11$$

$$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1 \quad c=\sqrt{5}$$



$f_1=(1,-2+\sqrt{5})$ $f_2=(1,-2-\sqrt{5})$

Conics - Ellipses

1. Find the equation of an ellipse with the following characteristics

   vertices (3, 1), (7, 1)

   vertices (5, 5), (5, -3)

$$\frac{(x-5)^2}{4}+\frac{(y-1)^2}{16}=1$$

2. Find the equation of an ellipse with the following characteristics

   Focus (2, 2), (10, 2)

   vertices (0, 2), (12, 2)

$$\frac{(x-6)^2}{36}+\frac{(y-2)^2}{20}=1$$