Chapter 3
Quadratic Equations and Complex Numbers

1. Solving Quadratic Equations
2. Complex Numbers
3. **Completing the Square**
4. Using the Quadratic Formula
5. Solving Nonlinear Systems
6. Quadratic Inequalities

3.3 - Completing the Square

Solve the quadratic using factoring

$$x^2 - 6x + 9 = 1$$

$$x = \{2, 4\}$$

$$x^2 - 30x + 225 = 121$$

$$x = \{-26, -4\}$$

3.3 - Completing the Square

Perfect square trinomial $x^2 - 16x + 64$

$$(x-8)(x-8) = (x-8)^2$$

Completing the Square - Sometimes you need to add a term to make a perfect square.

$$x^2 + bx \longrightarrow x^2 + bx + \left(\frac{b}{2}\right)^2 \longrightarrow \left(x + \frac{b}{2}\right)^2$$

$$x^2 + 8x \longrightarrow x^2 + 8x + 16 \longrightarrow (x+4)^2$$

3.3 - Completing the Square

Solve the equation by completing the square

$x^2 - 4x + 8 = 0$ $\quad x = 2 \pm 2i$

Practice

a) $x^2 + 8x - 5 = 0$ $\quad x = -4 \pm \sqrt{21}$

b) $6x(x + 2) = -42$ $\quad x = -1 \pm i\sqrt{6}$

3.3 - Completing the Square

Deriving quadratic formula

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3.3 - Completing the Square

Practice: Solve the equation by completing the square

22. $x^2 + 4x = 10$

23. $x^2 + 8x = -1$

25. $x^2 + 12x + 18 = 0$

26. $x^2 - 18x + 86 = 0$

28. $2k^2 + 16k = -12$

29. $3x^2 + 42x = -24$

3.3 - Completing the Square

Vertex Form

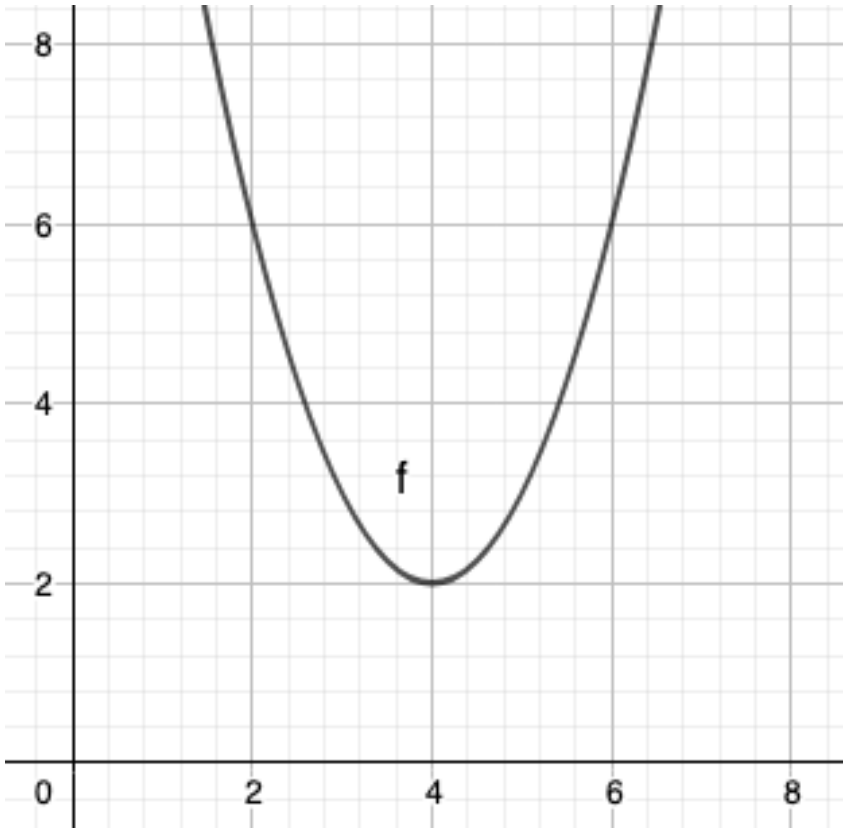
$$y = (x - h)^2 + k$$

(h, k) is the vertex of the parabola.

For example:

$$y = (x - 4)^2 + 2$$

Vertex
(4, 2)



3.3 - Completing the Square

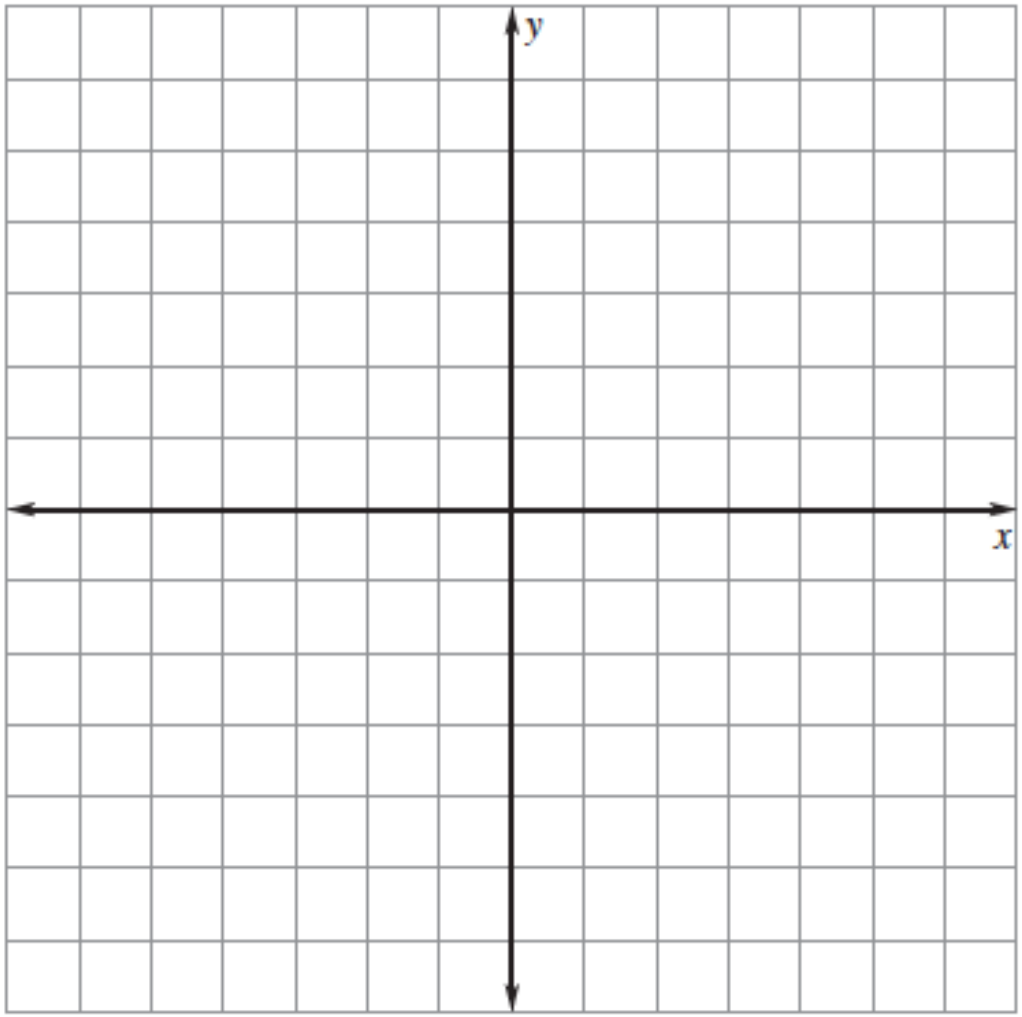
Solve for the Vertex Form $y = (x - h)^2 + k$

$y = x^2 + 6x + 4$ Vertex (-3, -5)

$y = (x + 3)^2 - 5$

$y = x^2 - 2x - 6$ Vertex (1, -7)

$y = (x - 1)^2 - 7$



3.3 - Completing the Square

Practice: Solve for the vertex form

41. $y = x^2 - 8x + 19$

42. $y = x^2 - 4x - 1$

43. $y = x^2 + 12x + 37$

44. $y = x^2 + 20x + 90$

45. $f(x) = x^2 - 3x + 4$

46. $g(x) = x^2 + 7x + 2$